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# A kinetic model for lutein production by the green microalga *Chlorella protothecoides* in heterotrophic culture

X-W Zhang, X-M Shi and F Chen

Department of Botany, the University of Hong Kong, Pokfulam Road, Hong Kong

Production of lutein by the green microalga *Chlorella protothecoides* grown heterotrophically in a fermentor using glucose as the carbon source and urea as the nitrogen source was investigated. An unstructured kinetic model was proposed to describe the microalgal culture system including cell growth, lutein formation, as well as glucose and nitrogen consumption. The inhibition potentials of biomass, product and substrates on growth and lutein formation were examined and incorporated into the kinetic model. Values of the kinetic model parameters were estimated. The resulting model predictions were in good agreement with the experimental results. The model can be helpful in scale-up, optimization and control of the *C. protothecoides* culture process, and can also be used as a guideline for similar microalgal cultivation systems.

Keywords: Chlorella protothecoides; heterotrophic culture; lutein production; unstructured kinetic model

### Introduction

Carotenoids have been proposed as cancer prevention agents, life extenders, and inhibitors of ulcer, heart attack and coronary artery disease [9]. Lutein is one of the most prominent carotenoids in human serum and in foods and has been used for pigmentation of animal tissues and products, as well as for coloration of foods, drugs and cosmetics [12]. Microalgae are one of the major sources of naturally occurring lutein [8].

At present, almost all industrial microalgal processes are based on open pond technology [3]. The inability to control environmental factors is a major unsolved problem which limits its development [4]. An alternative to overcome or minimize the problem with the photoautotrophic strategies is to develop heterotrophic culture technology using sugars or other organic compounds as the sole energy and carbon source(s). In this study, heterotrophic cultivation of *Chlorella protothecoides* was carried out in a fermentor using glucose as the carbon source and urea as the nitrogen source for lutein production.

Efforts have also been made to develop mathematical models to describe bioprocesses. Recently Bailey [2] made a comprehensive review of this subject. Structured models may be attractive due to their general applicability, but they suffer from the shortcomings of unknown or complicated mechanisms of regulation of cell culture, and often contain a large number of model parameters. Identification and estimation of these parameters are often difficult [23]. Therefore, unstructured kinetic models are the most frequently employed for modeling microbial systems because of their simplicity, and because they are adequate for technical purposes [11]. However, little information is available concerning kinetic models for heterotrophic microalgal culture systems with respect to cell growth, substrate consumption

and product formation, although the relationship between specific growth rate and substrate or light intensity was sometimes involved in investigations under photoautotrophic conditions of growth [10,14,19,20].

The aim of this work was to develop an unstructured kinetic model for lutein production using *C. protothecoides*. This model takes into account dependencies of growth and production on nutrients together with product inhibition and autoinhibition to analyze the dynamics of growth and metabolism of microalgae grown under heterotrophic culture conditions.

#### **Theoretical aspects**

#### Cell growth model

In the literature, two classes of models to describe the specific growth rate  $\mu$  are distinguished:

(1) Monotonic kinetics. The kinetics increase as a function of the substrate concentration  $C_s$ , such as the well-known Monod growth model [15]:

$$\mu = \frac{\mu_{\rm m} C_{\rm S}}{K_{\rm s} + C_{\rm S}} \tag{1}$$

where  $\mu_{\rm m}$  is the maximum specific growth rate.  $K_{\rm S}$  is the Monod saturation constant, a kinetic parameter which indicates how fast the maximum specific growth rate is reached. According to Eqn (1),  $\mu \rightarrow \mu_{\rm m}$  when  $K_{\rm S} \rightarrow 0$ . Unfortunately, the Monod model often fails to account for substrate inhibition of growth at higher substrate concentrations. To overcome the drawback, another model is employed:

(2) Non-monotonic kinetics. The non-monotonic is used, the Haldane model [1], as a function of the substrate concentration  $C_{\rm S}$ :

$$\mu = \frac{\mu_{\rm m} C_{\rm S}}{K_{\rm s} + C_{\rm S} + \frac{C_{\rm S}^2}{K_{\rm i}}}$$
(2)

Correspondence: Dr F Chen, Department of Botany, the University of Hong Kong, Pokfulam Rd, Hong Kong. E-mail: sfchen@hkuusa.hku.hk Received 28 January 1999; accepted 27 August 1999

where  $K_i$  is the inhibition constant. The smaller  $K_i$ , the larger the inhibition effect of the substrate.

It is obvious that both models examine only the effect of a single substrate and ignore the inhibition potential of other environmental factors such as the nitrogen source, the cell itself and the product.

Nitrogen is another essential substrate for cell growth, and here a Monod-type expression  $C_N/(K_{XN} + C_N)$  was employed to describe the effect of nitrogen. On the other hand, the expression  $(1 - C_p/C_{p_m})$  was used to describe product inhibition, as in glycerol fermentation [22]. Thus the Haldane model may be extended as follows:

$$\mu = \mu_{\rm m} \frac{C_{\rm S}}{K_{\rm S} + C_{\rm S} + \frac{C_{\rm S}^2}{K_{\rm xi}}} \frac{C_{\rm N}}{K_{\rm XN} + C_{\rm N}} \left(1 - \frac{C_{\rm P}}{C_{\rm P_{\rm m}}}\right)$$
(3)

where  $\mu$  is the specific growth rate (h<sup>-1</sup>),  $\mu_m$  is the maximum specific growth rate (h<sup>-1</sup>),  $C_P$ ,  $C_S$ ,  $C_N$  are the product concentration (mg L<sup>-1</sup>), the glucose concentration (g L<sup>-1</sup>) and the urea (nitrogen source) concentration (g L<sup>-1</sup>), respectively.  $C_{P_m}$  is the maximum product concentration (mg L<sup>-1</sup>).  $K_S$ ,  $K_{XN}$ ,  $K_{xi}$  are respectively the glucose saturation constant, the nitrogen saturation constant and the glucose inhibition constant of cell growth (g L<sup>-1</sup>).

#### Product formation model

The Luedeking–Piret kinetic model for product formation is Equation [13]:

$$\frac{\mathrm{d}C_{\mathrm{P}}}{\mathrm{d}t} = Y_{\mathrm{PX}} \frac{\mathrm{d}C_{\mathrm{X}}}{\mathrm{d}t} + \mu_{\mathrm{PX}} C_{\mathrm{X}} \tag{4}$$

The model states that the product formation rate of cells can be attributed to a growth-associated part and a nongrowth-associated part. Apparently, the model does not take into account the inhibition effects of glucose and nitrogen. In order to account for glucose inhibition of product forma-

tion, a Haldane-type expression  $\frac{C_{\rm S}}{K_{\rm PS} + C_{\rm S} + \frac{C_{\rm S}^2}{K_{\rm pi}}}$  was incorporated. Similarly, a Monod-type expression

porated. Similarly, a Monod-type expression  $C_{\rm N}/(K_{\rm PN} + C_{\rm N})$  was formulated accordingly to describe nitrogen inhibition as in modeling the growth of hybridoma cells [22]. Therefore, the Luedeking–Piret equation for product formation is extended as follows:

$$\frac{\mathrm{d}C_{\mathrm{P}}}{\mathrm{d}t} = \left(Y_{\mathrm{PX}} \frac{\mathrm{d}C_{\mathrm{X}}}{\mathrm{d}t} + \mu_{\mathrm{PX}}X\right) \frac{C_{\mathrm{S}}}{K_{\mathrm{PS}} + C_{\mathrm{S}} + \frac{C_{\mathrm{S}}^{2}}{K_{\mathrm{pi}}}} \frac{C_{\mathrm{N}}}{K_{\mathrm{PN}} + C_{\mathrm{N}}}$$
(5)

where  $C_X$ ,  $C_P$ ,  $C_S$ ,  $C_N$  are the same as above.  $K_{PS}$ ,  $K_{PN}$ and  $K_{pi}$  are the glucose saturation constant, the nitrogen saturation constant and the glucose inhibition constant of product formation (mg L<sup>-1</sup>), respectively.  $Y_{PX}$  is the yield coefficient of product formation due to cell growth in the early phase of culture (mg g<sup>-1</sup>). When in early cultivation, the cell concentration is lower, according to Eqn (4),  $dC_P/dt$   $\approx Y_{\rm PX} dC_{\rm X}/dt$ , ie  $Y_{\rm PX} = dC_{\rm P}/dC_{\rm X}$ .  $\mu_{\rm PX}$  is the specific formation rate of product in the later phase of culture (h<sup>-1</sup>). In fact, according to Eqn (4),  $\mu_{\rm PX} = dC_{\rm P}/Xdt$  when  $dC_{\rm X}/dt = 0$ , ie the concentration of cells is at the maximum value, the cells cease to grow.

#### Substrate consumption model

The most widely used substrate consumption model can be expressed as:

$$-\frac{\mathrm{d}C_{\mathrm{S}}}{\mathrm{d}t} = \frac{1}{Y_{\mathrm{XS}}}\frac{\mathrm{d}C_{\mathrm{X}}}{\mathrm{d}t} + \frac{1}{Y_{\mathrm{PS}}}\frac{\mathrm{d}C_{\mathrm{P}}}{\mathrm{d}t} + \mu_{\mathrm{SX}}C_{\mathrm{X}} \tag{6}$$

which is used as a glucose consumption model.  $C_X$ ,  $C_P$ ,  $C_S$ ,  $C_N$  are the same as above.  $Y_{XS}$  is the yield coefficient of cells on glucose (g g<sup>-1</sup>),  $Y_{PS}$  is the yield coefficient of product on glucose (mg g<sup>-1</sup>).  $\mu_{SX}$  is specific consumption rate of glucose (h<sup>-1</sup>) in the later phase of culture. In fact,  $\mu_{SX}$  =  $- dC_S/C_X dt$ , when  $dC_X/dt = dC_P/dt = 0$ , ie the concentrations of cell and product arrive at maximum values, the cell and product cease to accumulate, so the parameter is also considered to be related to maintenance functions of cells.

Similarly, the following equation was used to describe nitrogen consumption:

$$-\frac{dC_{\rm N}}{dt} = \frac{1}{Y_{\rm XN}} \frac{dC_{\rm X}}{dt} + \frac{1}{Y_{\rm PN}} \frac{dC_{\rm P}}{dt} + \mu_{\rm NX} C_{\rm X}$$
(7)

where  $C_X$ ,  $C_P$ ,  $C_S$ ,  $C_N$  are the same as above.  $Y_{XN}$  is the yield coefficient of cells on nitrogen (g g<sup>-1</sup>),  $Y_{PN}$  is the yield coefficient of product on nitrogen (mg g<sup>-1</sup>).  $\mu_{NX}$  is the specific consumption rate of nitrogen (h<sup>-1</sup>) in the later phase of culture, and can be considered to be a parameter related to maintenance requirements of cell metabolism like  $\mu_{SX}$ ; the analysis method is the same as for Eqn (6).

Consequently, an unstructured kinetic model (Eqns (3), (5), (6), (7)) for lutein production by heterotrophic cultivation of the microalga *Chlorella protothecoides* is proposed in this work. This model contains a total of 16 parameters, which have important physiological meanings; each one exhibits a process-state during the microalga cultivation.

Parameter estimations were performed using a Simplex Search Method; the principal idea of this method is to search the best value of the objective function in a space of points,  $P_i$  which represent the feasible solutions. The first point  $P_1$  is arbitrarily selected as the starting point (base point). The second point  $P_2$  is chosen and compared with  $P_1$ . If  $P_2$  is found to be a better solution than  $P_1$ , then  $P_2$ is selected as the new base point; if not,  $P_1$  stays as the base point. This process is continued until the best operating point is found. The details were described previously [23]. The starting values of model parameters were determined based on physiologically meaningful ranges. The fitting result was evaluated by the root-square residuals:

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where *E* is the fitting error,  $Y_{exp}$  and  $Y_{cal}$  are experimental and calculated values, respectively and *Q* is data number.

#### Materials and methods

### Microalgal strain and medium

*C. protothecoides* CS-41 was obtained from the CSIRO Marine Laboratory, Hobart, Australia. The modified Basal medium supplemented with 40 g  $L^{-1}$  glucose and 3.6 g  $L^{-1}$  urea was used [16].

#### Cultivation

Sterilized medium (121°C, 15 min) was inoculated with 5% exponentially growing inocula. Heterotrophic cultivation of *C. protothecoides* CS-41 was initially carried out in a 250-ml Erlenmeyer flask containing 150 ml medium at  $28 \pm 1^{\circ}$ C under continuous shaking (180 rpm) in the dark. Further heterotrophic cultivation was performed in a 3.7-L fermentor (Bioengineering AG, Wald, Switzerland) containing 2.5 L medium. The cultivation conditions in the fermentor were controlled as follows: pH 6.6 ± 0.1; temperature 28°C; agitation 480 rpm; dissolved oxygen concentration 50% saturation.

## Analytical method

The dry cell weight concentration was determined according to Chen *et al* [6]. Glucose and nitrate concentrations were determined as previously described [18]. Lutein concentration was determined according to Shi and Chen [16,17]. Specific growth rate,  $\mu$ , was determined according to Chen and Johns [5].

## **Results and discussion**

The experimental results for heterotrophic production of biomass and lutein by *C. protothecoides* in a 3.7-L fermentor are shown in Figures 1–4. By fitting all the experi-



**Figure 1** Prediction (----) of growth of *C. protothecoides* under heterotrophic conditions using a kinetic model (Equation 8) compared to the experimental data  $(\blacksquare)$ .



**Figure 2** Prediction (——) of lutein concentration in the culture of *C. protothecoides* under heterotrophic conditions using a kinetic model (Equation 9) compared to the experimental data ( $\blacksquare$ ).



**Figure 3** Prediction (——) of glucose consumption in the culture of *C. protothecoides* under heterotrophic conditions using a kinetic model (Equation 10) compared to the experimental data ( $\blacksquare$ ).

mental data, the following kinetic models proposed in this work were given:

$$\frac{dC_{\rm X}}{dt} = \frac{0.0527C_{\rm S}C_{\rm X}}{0.1218 + C_{\rm S} + \frac{C_{\rm S}^2}{110.0605}}$$

$$\frac{C_{\rm N}}{0.3070 + C_{\rm N}} \left(1 - \frac{C_{\rm P}}{76.2734}\right)$$
(8)

$$\frac{dC_{\rm P}}{dt} = \left(2.3584 \ \frac{dC_{\rm X}}{dt} + 0.0265C_{\rm X}\right) \tag{9}$$

$$\frac{C_{\rm S}}{0.7536 + C_{\rm S} + \frac{C_{\rm S}^2}{120.1358}} \frac{C_{\rm N}}{0.0546 + C_{\rm N}}$$

$$-\frac{dC_{\rm S}}{dt} = \frac{1}{0.5886} \frac{dC_{\rm X}}{dt} + \frac{1}{2.2878} \frac{dC_{\rm P}}{dt} + 0.0128C_{\rm X}$$
(10)



**Figure 4** Prediction (---) of nitrogen consumption in the culture of *C. protothecoides* under heterotrophic conditions using a kinetic model (Equation 11) compared to the experimental data ( $\blacksquare$ ).

$$-\frac{\mathrm{d}C_{\mathrm{N}}}{\mathrm{d}t} = \frac{1}{6.3871} \frac{\mathrm{d}C_{\mathrm{X}}}{\mathrm{d}t} + \frac{1}{25.8963} \frac{\mathrm{d}C_{\mathrm{P}}}{\mathrm{d}t} + 0.0013C_{\mathrm{X}}$$
(11)

The fitting errors (E) were 0.5877, 1.4825, 0.9994, 0.2258 for cell growth, lutein formation, glucose consumption and nitrogen consumption, respectively. The calculated results for Eqns (8)–(11) are also shown in Figures 1–4.

Equation (8) revealed that the maximum specific growth rate was reached more slowly for consumption of glucose than for consumption of nitrogen, because the velocity indication parameter  $K_{\rm S} = 0.1218$  g L<sup>-1</sup> was smaller than  $K_{\rm NX}$ = 0.3070 g L<sup>-1</sup>. This suggested that glucose might be a limiting substrate. In fact, in the cultivation, glucose was used up by the microalga ahead of the nitrogen. On the other hand, based on the inhibition parameter  $K_{xi}$  = 110.0605 g  $L^{-1}$ , it demonstrated that glucose exhibited an inhibitory effect on cell growth to some extent. Moreover, according to Eqn (8) the maximum lutein yield was  $C_{\rm pm} =$ 76.2734 mg  $L^{-1}$ , and the maximum specific growth rate was  $\mu_{\rm m} = 0.0527$  (h<sup>-1</sup>), while the experimental values were 83.81 mg  $L^{-1}$  and 0.0425 (h<sup>-1</sup>), respectively. This showed a satisfactory fit between the model predictions and experimental data.

From Eqn (9) the yield coefficient of product formation due to cell growth ( $Y_{PX}$ ) and specific formation rate of product in the early phase of cell growth ( $\mu_{PX}$ ) can be calculated to be 2.3584 (mg g<sup>-1</sup>) and 0.0265 (h<sup>-1</sup>), respectively. While the corresponding values determined by experimental data were 2.6289 mg g<sup>-1</sup> in the early phase of culture (t = 20 h) and 0.0232 (h<sup>-1</sup>) in the later phase of culture (t = 140 h) (cell growth approaches the maximum value 19.6 g L<sup>-1</sup> at 142 h). These were again in good agreement. Also, based on the inhibition constant  $K_{pi} = 120.1358$  mg L<sup>-1</sup>, glucose had an inhibitory effect on product formation.

Furthermore, with Eqns (10) and (11) the following information may be obtained: the yield coefficient of cells on glucose  $Y_{\rm PS}$  is 0.5886 g g<sup>-1</sup>, the yield coefficient of produt on glucose  $Y_{\rm PS}$  is 2.2878 mg g<sup>-1</sup>, the specific consumption rate of glucose  $\mu_{\rm SX}$  in the later phase of culture is 0.0128 (h<sup>-1</sup>), while the experimental values of correspond-

 $\label{eq:table_$ 

Model	Parameter	Error	Reference
Monod model (Equation 1)	$\mu_{\rm m} = 0.0147 \ ({\rm h}^{-1})$ $K_{\rm s} = 3.1618 \ ({\rm g \ L}^{-1})$	1.7896	[15]
Haldane model (Equation 2)	$\begin{array}{l} \mu_{\rm m} = 0.0150 ~({\rm h^{-1}}) \\ K_{\rm S} = 3.1618 ~({\rm g}~{\rm L^{-1}}) \\ K_{\rm i} = 594.1771 ~({\rm g}~{\rm L^{-1}}) \end{array}$	1.8308	[1]
Equation 3	$\begin{array}{l} \mu_{\rm m} = 0.0527 ~({\rm h}^{-1}) \\ K_{\rm S} = 0.1218 ~({\rm g}~{\rm L}^{-1}) \\ K_{\rm XI} = 110.0605 ~({\rm g}~{\rm L}^{-1}) \\ K_{\rm XN} = 110.0605 ~({\rm g}~{\rm L}^{-1}) \\ C_{\rm pm} = 76.2734 ~({\rm mg}~{\rm L}^{-1}) \end{array}$	0.5877	This work

ing parameters were 0.48 g g<sup>-1</sup>, 2.08 mg g<sup>-1</sup>, 0.0159 (h<sup>-1</sup>) in the later phase of culture (t = 120 h), respectively. The yield coefficient of cells on nitrogen,  $Y_{XN}$ , is 6.3871 g g<sup>-1</sup>, the yield coefficient of product on nitrogen  $Y_{PN}$ , is 25.8963 mg g<sup>-1</sup> and the specific consumption rate of nitrogen,  $\mu_{NX}$ , in the later phase of culture is 0.0013 (h<sup>-1</sup>) while the experimental values of corresponding parameters were 5.44 g g<sup>-1</sup>, 23.2806 mg g<sup>-1</sup> and 0.0016 (h<sup>-1</sup>) in the later phase of culture (t = 120 h), respectively. Consistency was achieved between the predicted data and experimental results.

High cell-density culture of *Chlorella* in a batch system requires comparatively high concentrations of nitrogen in the medium. It was reported that urea at 2.0 g L<sup>-1</sup> (0.14 M) nitrogen in the medium had ill effects on the growth of *C. pyrenoidosa* [7]. A similar effect was observed by [10] when *C. vulgaris* was cultivated in the presence of nitrate at a concentration as high as 0.097 M. In contrast, a high concentration (10 g L<sup>-1</sup>) of nitrate and urea was also used in the cultivation of *C. pyrenoidosa*, with no ill effect found [21]. In this study, there was no inhibitory effect observed on growth of *C. protothecoides* when the concentration of urea was maintained at 3.6 g L<sup>-1</sup> (0.12 M) nitrogen in the medium. These observations show that the tolerance of various *Chlorella* strains towards the concentration of nitrogen sources might be different.

Finally, to compare the simulations, the classical models such as the Monod (Eqn (1)), the Haldane (Eqn (2)) and the Luedeking–Piret (Eqn (4)) models were employed to fit the experimental data. The parameters and fitting errors for different models estimated with the data of cell growth or product formation are given in Tables 1 and 2.

 Table 2
 Parameter values and fitting errors for different models of product formation

Model	Parameter	Error	Reference
Luedeking–Piret model (Equation 4)	$\mu_{\rm PX} = 0.0229 \ ({\rm h}^{-1})$ $Y_{\rm PX} = 1.7441 \ ({\rm mg \ g}^{-1})$	1.6294	[13]
Equation 5	$\begin{array}{l} \mu_{\rm PX} = 0.0265 \ (h^{-1}) \\ Y_{\rm PX} = 2.3584 \ (mg \ g^{-1}) \\ K_{\rm PS} = 0.7536 \ (mg \ L^{-1}) \\ K_{\rm Pi} = 120.1358 \ (mg \ L^{-1}) \\ K_{\rm PN} = 0.0546 \ (mg \ L^{-1}) \end{array}$	1.4825	This work

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Equations (1) and (2) describe the cell growth data with the average errors (E) 1.7896 and 1.8308, respectively, larger than that of Eqn (3) proposed in this work (E = 0.5877). The fitting error for Eqn (4) to describe lutein production was 1.6294, larger than that of Eqn (5) proposed in this work (E = 1.4825). Furthermore, all these models do not take into account the effects of nitrogen- and productinhibition. Therefore, the kinetic model for lutein production by microalga culture (Eqns (3), (5)–(7)) gave a superior fit over the other classical models. What is most important is that a number of kinetic parameters of physiological significance can be determined, and can be used as guidelines for other microalgal culture systems.

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